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Evaluation of the Kelvin Impulse on Single Bubble Shock-Induced Collapse Near Flexible Boundaries

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Abstract: This present paper provides an estimation of the Kelvin Impulse by utilizing point-source equation developed by Blake *et al.* then incorporating it to numerical simulations. A 2D axisymmetric model with 60 mm × 60 mm size was established to model water tank with a plate fixed at the edge. A single stationary air bubble was placed with various stand-off parameter γ near the plate where boundary type varies from free surface, flexible, and rigid. To estimate the Kelvin Impulse, gauges were installed on the surface of the air bubble. First, free surface and rigid case with various δ and γ values were simulated to predict jet direction. Results shows that jet direction with various $\delta\gamma$ value predicted from simulation exhibit similar behavior with previous researcher's result. Therefore, the Kelvin Impulse value of γ=1 with three types of materials as boundaries; titanium, polyethylene, and rubber which represents high, medium, and low flexural rigidity were calculated in the same manner. It is found that the Kelvin Impulse for rubber and polyethylene were negative with jet directions away from the boundary, which behaves similar as free surface. In titanium, the Kelvin Impulse was positive and the jet towards the boundary, which was similar as rigid boundary.

Keywords: cavitation damage; Kelvin Impulse; bubble dynamics

1. Introduction

In the middle of operation, hydraulic machinery usually suffers from severe vibration, noise, and cavitation damage. One of the examples is hydro turbine with huge capacity of 1000 MW per unit in China, which has been developed for sustainable energy supply in the world, has cavitation damage as its main maintenance problem [1]. Cavitation is a formation of small vapor-filled cavities or called "bubbles" due to rapid change of pressure. The unstable bubble will generate intense shock wave and micro jet which can damage nearby structure.

Benjamin and Ellis [2] were the first who provided experimental proof of micro-jet formation. Later, it was theoretically approached by Plesset and Chapman [3]. As a response to surrounding pressure wave, the initially stationary bubble with maximum radius starts to oscillate. Then, liquid start to penetrates the bubble and flows through it towards the boundary, forming a micro-jet [4]. The direction of this micro-jet can be either towards or away from the boundary. Blake *et al*. [5] utilized concept of the Kelvin Impulse to calculate force emitted from bubble, hence gross predicting bubble motion and also micro-jet direction. However, theoretical solution only applies into rigid, free surface, membrane, and inertial boundary, whereas in real life, boundary has finite value of stiffness i.e. flexible. This paper aims to apply bubble motion data obtained from numerical simulation into point source equation developed by Blake *et al*. to further estimate jet direction and the Kelvin Impulse of shock-induced collapse near flexible boundaries.

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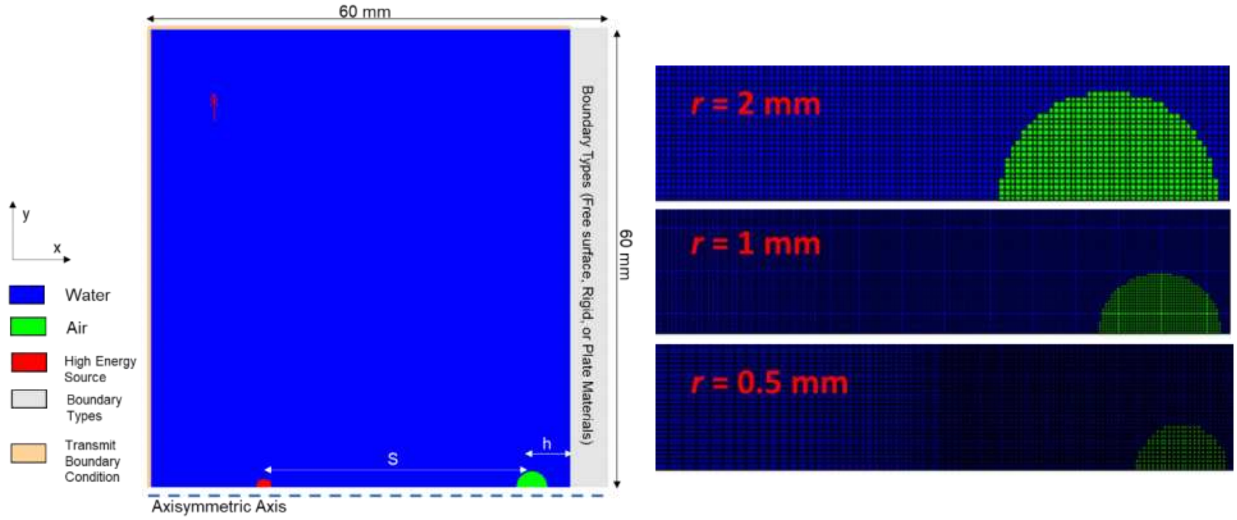
2. Materials and Methods

Numerical hydrocode simulation on ANSYS AUTODYN in Fig. 1a depicts 2D axisymmetric model with boundary placed at the top of water surface and fixed at the edge. Bubble with radius r was placed at the axisymmetric axis with distance h from boundary (free surface, rigid, and plate materials). A high energy source of 3×10^5 kJ was placed at distance S from bubble. This high energy source generated shockwave which induced the bubble to collapse. Mesh type for fluid is Eulerian whereas for plate materials is Lagrangian. Shock EOS was used to model fluid and plate materials. Mesh refinement near bubble area was done in order to save computational time, as shown on Fig. 1b. Note that total number of elements varies depend on bubble size, for instance, finest mesh size was used for smallest bubble size r = 0.5 mm.



(**a**) (**b**)

Figure 1. (**a**) Numerical set-up for air bubble and shockwave in the vicinity of rigid, free surface, or flexible boundary; (**b**) Mesh grid variations for bubble r = 2 mm, 1 mm, and 0.5 mm.

In order to extract force emitted from bubble collapse or the Kelvin Impulse, a discretization from point-source equation developed by Blake *et al*. [5] was conducted. It is defined as follows:

$$F_x(t) = \rho\pi \sum_{t=0}^{t} r(u^2 - v^2)\Delta r \tag{1}$$

where F_x is force emitted from bubble collapse in x direction, u is in x-direction, v is velocity in y direction, and $r = (y^2 + z^2)^{1/2}$. Due to axisymmetric condition, $z = 0$, then r is equal to y. Hence, Δr is the difference between y position on t and t+1. Those necessary values were extracted from gauges that installed on each nodes of bubble surface. The Kelvin Impulse or I_x was obtained by simply integrating F_x through time from the beginning until collapse time T_c. The direction of micro-jet was considered from I_x value as follows:

$$I_x = \sum_{t=0}^{T_c} F_x \Delta t \quad ; \quad I_x = \begin{cases} < 0 : \ jet\ towards\ boundary \\ > 0 : \ jet\ away\ from\ boundary \\ \approx 0 : neutral\ jet\ (inward\ equatorial\ jet) \end{cases} \tag{2}$$

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3. Results

3.1. Kelvin Impulse of Rigid and Free Surface Boundaries

A rigid boundary is supposed to have infinite stiffness while free surface boundary has zero stiffness. The Kelvin Impulse theory derived by Blake *et al*.[5] from point source velocity potential on bubble surface stated that χ parameter on rigid boundary equals to -1, while for free surface equals to 1. Therefore, as seen in Eq. (2), for rigid boundary jet direction will be towards boundary while for free surface it will be away from boundary. However, a later study from Blake and Taib [6-7] which incorporated Boundary Element Method on bubble collapse near rigid and free surface proves that the direction of jet change as stand-off parameter γ and buoyancy parameter δ varies. Those parameters defined as follows:

$$\gamma = h/r \quad and \quad \delta = (\rho g r/\Delta p)^{1/2} \tag{3}$$

In this paper, three values of δ and γ parameters were simulated for each free surface and rigid boundaries. Results shown that $I_x < 0$ or jet direction towards boundary on $\delta\gamma = 1.30$ for free surface and $\delta\gamma = 0.26$ for rigid. Conversely, jet direction away from boundary or $I_x > 0$ on $\delta\gamma = 0.26$ for free surface and $\delta\gamma = 1.30$ for rigid boundary.

Table 1. Kelvin Impulse I_x value for free surface and rigid boundary.

Boundary Type	δ	γ	$\delta\gamma$	T_c (ms)	I_x (ms)
Free Surface	0.26	5	1.30	0.064	-0.035
	0.13	2	0.26	0.069	0.010
Rigid	0.26	5	1.30	0.087	10.50
	0.13	2	0.26	0.064	-2.00

This result shows that the discretization approach of point-source equation (1) on shock-induced bubble collapse simulation qualitatively agrees with what Blake and Taib found using both of BEM and experiment, which stated that for free surface [6], on $\delta\gamma > 0.442$ jet direction will change from away to towards boundary. Conversely for rigid [7], on $\delta\gamma > 0.442$ jet direction will change from towards to away from boundary.

3.2. Kelvin Impulse of Flexible Boundaries

A flexible boundary is known to have a finite value of flexural rigidity and will affect motion when subjected into a load. The χ parameter introduces on [5] which determine jet direction of bubble collapse has no relation on mechanical properties and only applies for free surface, membrane, inertial in rigid boundary. Therefore, three materials which represents low, medium, and high plate rigidity D were simulated as 1 mm thickness flexible boundary plate fixed at edge. The results as follows:

Table 2. Kelvin Impulse I_x value for flexural boundaries.

Material Types	D (Pa.s/mm³)	δ	γ	$\delta\gamma$	T_c (ms)	I_x (ms)
Titanium	8.7×10^{-5}	0.13	1	0.13	0.054	-0.0320
Polyethylene	0.012	0.13	1	0.13	0.070	0.0044
Rubber	1.481	0.13	1	0.13	0.054	0.0040

Results shown that for Polyethylene and Rubber, $I_x > 0$ or jet direction were away from boundary. Conversely for Titanium, jet direction was towards boundary because $I_x < 0$. The value of I_x for Titanium is 10 times larger than Polyethylene and Rubber. Bubble deformation phase of three materials from $t = 0$ until T_c can be seen on Figs. 2a, 2b, and 2c. Flexural Rigidity D in logarithmic form of all materials were mapped into the Kelvin Impulse I_x as shown in Fig. 2d.

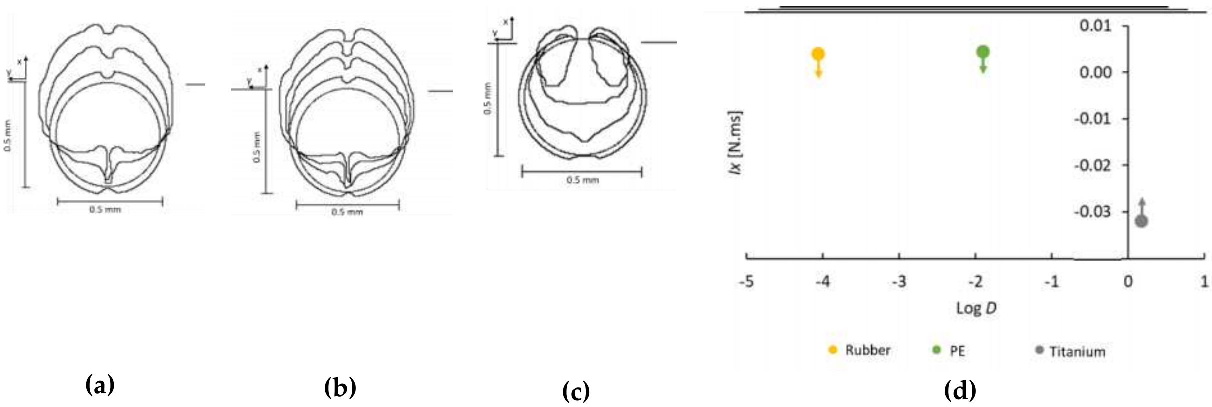

Figure 2. Bubble deformation from $t = 0$ until collapse time T_c for (b) Rubber, (c) Polyethylene, and (d) Titanium for $\delta\gamma = 0.13$; (d) the Kelvin Impulse I_x and log D mapped for Rubber, Polyethylene, and Titanium. Arrow represents jet direction from flexible boundary oriented at the top.

Free surface is considered to have zero stiffness while rigid has infinite stiffness. Therefore, if both boundaries were mapped into Fig. 2d, free surface would be on the far left and rigid would be on far right. Considering that for $\boldsymbol{\delta\gamma < 0.442}$ jet direction is away from boundary or $\boldsymbol{I_x > 0}$ for free surface [6] and jet direction towards boundary or $\boldsymbol{I_x < 0}$ for rigid [7], it can be stated that both Rubber and Polyethylene has similar trends to free surface while Titanium follows rigid.

This result shows that for low to medium *D*, the tendency of jet direction is away from boundary meanwhile for medium to high *D*, jet direction towards boundary. Katz [8] found anti-cavitation properties on soft rubber coating where there was no cavitation pit formed. This might mean that the anti-cavitation properties were jet direction away from boundary, which occurred due to low stiffness of soft rubber material. Therefore, more research on variating material stiffness is needed to further explore these phenomena, which may be used in the future to create a material that can prevent cavitation damage.

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