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Study of flow problems a super-cavitating body under conditions of significant accelerations of motion

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Abstract: The research data are presented in relation to the solution of the problem of gaining a high speed for motion a body in a supercavity with gas injection, starting with low initial speeds of movement. Two main possibilities of control by the cavity during uniformly accelerated motion are determined, ensuring the preservation of its shape and dimensions close to the shape and dimensions of a stationary cavity at the initial moment of motion.

Key words: Supercavitation, acceleration, gas injection.

1. Introduction

The maximum efficiency for body motion in the supercavity is achieved under fits most closely into the cavity with the smallest possible clearance. At the same time, with a significant change in the movement speeds, the dimensions of the body and the cavity can differ significantly. The purpose of this work is to study the possibilities of controlling an unsteady axisymmetric supercavity, which ensures that its dimensions and shape remain close to a stationary cavity during accelerated motion. Such control promotes more favorable conditions for body movement in the cavity, but does not provide stability of movement, what are the subject of separate consideration.

2. Methods for calculating non-stationary supercavities

Investigations are carried out on the basis of equations for the shape of an unsteady cavity [1], obtained on the basis of the Slender Body Hydrodynamics. These equations are a mathematical formulation of the well-known "principle of independence of cavity expansion" [2]. The system of equations for calculating an unsteady axisymmetric cavity in a coordinate system, associated with a stationary fluid is defined as (1, 2) [1]:

$$\text{a)}\ \frac{\partial^2 R^2(x,t)}{\partial t^2}+\frac{2\Delta P(x,t)}{\rho\mu}=0\,,\ \text{b)}\ \left.\frac{\partial R^2}{\partial t}\right|_{t=t_n(x)}=R_n(x)U_n(x)\sqrt{\frac{2\left[c_d(x)-k\sigma_n(x)\right]}{k\mu}}\,,\ \text{c)}\ R_n=R_{nt}(t)\big|_{t=t_n(x)}=R_n^2(x)\,, \tag{1}$$

$$\text{a)}\ U=U(t)\big|_{t=t_n(x)}=U_n(x)\,,\ \text{b)}\ c_d=c_{dt}(t)\big|_{t=t_n(x)}=c_d(x)\,,\ \text{c)}\ \sigma_n(x)=0.5\left[\Delta P(x,t)/\rho U^2(t)\right]_{t=t_n(x)} \tag{2}$$

Here: U, R_n, c_d - speed of movement, radius, drag coefficient of the cavitator which can depend on time t, $R=R(x,t)$ non- steady axisymmetric super cavity form, $\Delta P=P_\infty(x)-P_c(t)$ the pressure difference in the flow and in the cavity, $\Delta P_n(t)$ - at the section behind cavitator, ρ - water mass density, $x_n(t)$ - the cavitator law of motion , $t_n(x)$ is the inverse function of the law $x_n(t)$. The quantities μ, k are slowly varying functions, mainly from the elongation of the cavity $\lambda=L/2R_k$, L is the cavity length, R_k is the maximal cavity radius, μ characterizes the inertia of the cavity sections, k - the energy transfer along its cross sections. The results of calculating the asymptotic dependences for μ and k Eq. (3a, 3d, 4a) [1] - dotted line, together with their improved on the basis of nonlinear numerical calculation [3] approximations Eq. (3b, 3c, 4b, 4c) are presented in Figures 1-3.

$$\text{a)}\ \mu=\ln\frac{\lambda}{\sqrt{e}}:\ \text{b)}\ \mu\simeq 0.5\ln\left(\frac{\lambda^2+7}{e}\right),\ \text{c)}\ \mu\simeq\ln\sqrt{\frac{1}{e\sigma}\ln\left(\frac{2}{\sigma}+10\right)};\ \text{d)}\ \sigma=\frac{2\ln\left(\lambda/\sqrt{e}\right)}{\lambda^2}=\frac{2\mu}{\lambda^2}:\ \text{e)}\ \lambda=\sqrt{\frac{2\mu}{\sigma}} \tag{3}$$

$$\text{a)}\ k\simeq 1-\frac{2\ln\left(2/\sqrt{e}\right)}{\ln\lambda^2}:\ \text{b)}\ k\simeq 1-\frac{2\ln\left(2/\sqrt{e}\right)}{\ln\left(0.8\lambda^2+35\right)},\ \text{c)}\ k\approx 1-\frac{2\ln\left(2/\sqrt{e}\right)}{\ln\left(4/\sigma+18\right)} \tag{4}$$

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Figure 1. **(a)** Calculations μ : ----- Eq. (3a) , ——— Eq. (3b), • • • • nonlinear calculations [3]; **(b)** Calculations $\lambda(\sigma)$: ----- Eq. (3d), ——— Eq. (3c- 3e), • • • • numerical calculations [3]

Figure 2. **(a)** Calculations k : ----- Eq. (4a), ——— Eq. (4b), • • • • numerical calculations [3]; **(b)** The shape of the stationary cavity forward part: ——— experiments date [2], ▲▲▲▲ - numerical calculations for $\sigma = 0.01$ [3], Calculations : ━━━ Eq.(6b) for $\sigma = 0.01$, -△-△-△-△- Eq. (6a) $\sigma = 0.04$, ----- Eq (6a), $\sigma = 0.004$.

The accuracy of equations (1, 2) is illustrated by the solution for a stationary cavity based on the stationary version of these equations (6). The results of calculating this solution (6) for the cavity behind the disk at the number of cavitation are presented by the figure 3, $\bar{R} = R / R_n$, $\bar{x} = x / R_n$:

$$\text{a) } \frac{d^2R^2}{dx^2} + \frac{\sigma}{\mu} = 0\ ,\ \left.\frac{dR^2}{dx}\right|_{x=0} = R_n\sqrt{\frac{2(c_d - k\sigma)}{k\mu}}\ ,\ \left.R^2\right|_{x=0} = R_n^2 :\ \bar{R} = \sqrt{1 + \sqrt{\frac{2(c_d - k\sigma)}{k\mu}}\bar{x} - \frac{\sigma}{2\mu}\bar{x}^2}\ ,\ \text{b) } \bar{R} = (1 + 4.5\bar{x})^{\frac{1}{3.3}} \qquad (6)$$

Calculation results based on Eq. (6) Figures 2 (b) and 3 indicate a significant not accurate in calculating of the cavity shape in a small region directly near the disk with the possibility to use there approximation - Eq. (6b):

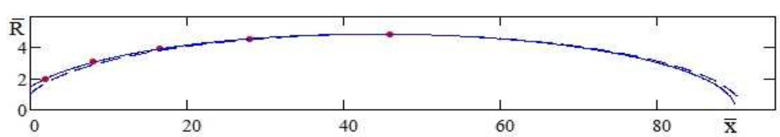

Figure 3. Stationary axisymmetric cavity behind the disk at $\sigma = 0.04$

----- Eq. (6), ——— Eq. (7c), • • • • nonlinear numerical calculations [3].

The transformation of the solution (6a) in the form (7c) in the calculation, starting from the section $\bar{x} = 2$, $\bar{R} = 2$ Eq. (7a, 7b) leads, as can be seen in Figure 3, to the radical improvement of the solution as a whole.

$$\text{a) } \frac{dR^2}{dx} = R_n \left.\sqrt{\frac{2(c_d - 4k\sigma)}{k\mu}}\right|_{x=2R_n}\ ,\ \text{b) } \left.R^2\right|_{x=2R_n} = 4R_n^2 :\ \text{c) } \bar{R} = \sqrt{4 + \sqrt{\frac{2(c_d - 4k\sigma)}{k\mu}}(\bar{x} - 2) - \frac{\sigma}{2\mu}(\bar{x} - 2)^2} \qquad (7)$$

Solution of problem (1) for a non stationary cavity is presented in the form of the universal integral:

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$$R^2(x,t) = R_n^2(x) + 2R_n(x)U_n(x)\sqrt{\frac{c_d(x)-k\sigma_n(x)}{2k\mu}}(t-t_n(x)) - \frac{1}{\rho\mu}\left[[\Delta P_o + \Delta P_x(x)](t-t_n(x))^2 + 2\int_{t_n(x)}^{t}\left(\int_{t_n(x)}^{t}\Delta P_{xt}(x,s)ds\right)\right] \quad (8)$$

2. Optimization of the cavity shape in the process of uniformly accelerated motion

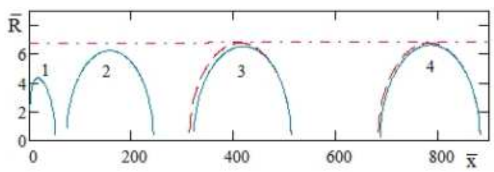
Taking into account the dimensionlessness of quantities (9) with respect to ρ , a , R_n , a - acceleration, σ_{nx} , σ_{nx} , corresponds to the time $\bar{t} = \bar{t}_n(\bar{x})$, the solution of equations (1) for cavity behind the disk at the $\Delta P = \text{const}$ in the form of (10) determines the shape of the cavity with essential increase in its size during motion $(c_d - k\sigma) \simeq c_{do}$:

$$\text{a) } \bar{x}_n(t) = \frac{\bar{t}^2}{2}, \text{ b) } \bar{t}_n(\bar{x}) = \sqrt{2\bar{x}}, \text{ c) } \bar{U}_n(\bar{t}) = \bar{t}, \text{ d) } \bar{U}_x(\bar{x}) = \sqrt{2\bar{x}}, \text{ e) } \sigma_{nt} = \frac{2\Delta\bar{P}_t(\bar{t})}{\bar{t}^2}, \text{ f) } \sigma_{nx} = \frac{\Delta\bar{P}_x(\bar{x})}{\bar{x}} \quad (9)$$

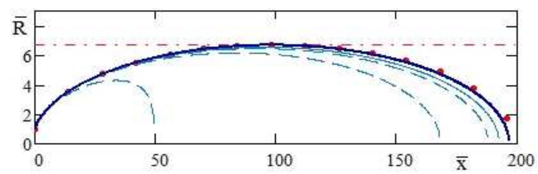
$$\text{a) } \bar{R}^2 \approx 1 + \sqrt{2\bar{x}}\sqrt{\frac{2c_{do}}{k\mu}}\left[\bar{t} - \sqrt{2\bar{x}}\right] - \frac{\Delta\bar{P}}{\mu}\left[\bar{t} - \sqrt{2\bar{x}}\right]^2, \text{ b) } \sigma_{nt} = \frac{2\Delta\bar{P}}{\bar{t}^2} \quad (10)$$

For $\sigma_{nt} = \sigma_a = \text{const}$ the solution (11a) determines the shape of a non-stationary cavity close to stationary during accelerated in motion starting from $\bar{t}_o = \bar{U}_o$ - Eq. (11b). The calculation results here are illustrated in Figure. 4.

$$\text{a) } \bar{R}^2 = 1 + \sqrt{\frac{2c_{do}}{k\mu}}\sqrt{2\bar{x}}\left(\bar{t} - \sqrt{2\bar{x}}\right) - \frac{\sigma_a}{\mu}\left(\frac{1}{12}\bar{t}^4 + \bar{x}^2 - \frac{2\sqrt{2}}{3}\bar{t}\,\bar{x}^{3/2}\right), \text{ b) } \bar{U}_o > \frac{6\kappa_s}{\sqrt{\sigma_a}} \quad (11)$$



(a)



(b)

Figure 4. The results of calculating Eq.(11a) a non-stationary cavity behind the disk when the pressure in the cavity changes, ensuring that its shape is close to the stationary one in the process of accelerated motion. The results are given for case of $\sigma_a = 0.02$ in two systems of coordinates: **(a)** cavitator is moving in motionless fluid: ——--- 1 - $\bar{t} = 10$, 2 - $\bar{t} = 22$, 3 - $\bar{t} = 32$, 4 - $\bar{t} = 42$, ---- Stationary cavity at $\sigma_a = 0.02$; **(b)** Fluid is moving, cavitator is motionless: ---- $\bar{t} = 10$, $\bar{t} = 22$, $\bar{t} = 32$, ——--- $\bar{t} = 42$, ━━━ stationary cavity at $\sigma_a = 0.02$.

3. Intensity of gas injection during accelerated motion, forming a cavity close to stationary one

Taking into account the designations $dm/dt = m^\bullet$, the calculation of the gas injection intensity $m_{in}^\bullet(t+t_o)$ is determined by the equations (12-13) based on the gas mass conservation equation (12a) within the framework of the isentropic ideal gas flow. Here $m_c^\bullet(t+t_o)$, $m_{out}^\bullet(t+t_o)$ are the intensities of the change in the mass of gas in the cavity and its entrainment from the cavity, P_c, ρ_c , - are the pressure and mass density of the gas in the cavity at their values P_{co} , ρ_{co} at $t = t_o$. Value ρ_{co} is defined on the base of equation of ideal gas for temperature $t_{co}^o = 15^o$ Celsius . For calculation the volumetric gas entrainment $Q_{out}^\bullet$ dependence of the publication [4] is using for $c_q = 0.017$ value in Eq. (13). κ_c - coefficient of free volume of the cavity outside the body in the cavity.

$$\text{a) } m_c = \rho_c(t+t_o)Q_c(t+t_o),\ m_c^\bullet(t+t_o) = m_{in}^\bullet(t+t_o) - m_{out}^\bullet(t+t_o),\ \text{b) } P_c = P_\infty - \frac{\sigma_a\rho}{2}\left(a(t+t_o)\right)^2 \quad (12)$$

$$\rho_c = \frac{\rho_{co}}{P_{co}^n}P_c^n,\ \text{b) } \bar{Q}_{in}^\bullet(t) = \left[c_q\pi\frac{c_d}{\sigma_a}\sqrt{\frac{1}{\sigma_a}\ln\frac{1}{\sigma_a}}\left(\bar{t}+\bar{t}_o\right) - \pi n\left(\frac{4}{3}\frac{\kappa_c}{\sigma_a^2}\frac{c_d\sqrt{2\mu c_d}}{k\sqrt{k}}\right)\left(\frac{\sigma_a\left(\bar{t}+\bar{t}_o\right)}{\bar{P}_\infty - 0.5\sigma_a\left(\bar{t}+\bar{t}_o\right)^2}\right)\right],\ \bar{m}_{in}^\bullet = \bar{\rho}_c\bar{Q}_{in}^\bullet \quad (13)$$

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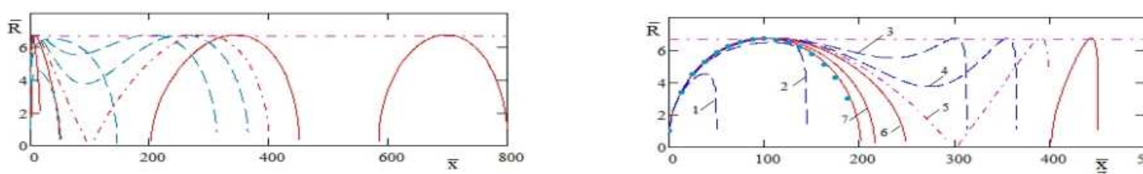
(a) (b)

Figures 5. a) Limitations of the range of velocities at the initial moment $\bar{U}_o = \bar{U}_o(\sigma)$, corresponding to the proximity of the forms of non-stationary and stationary cavities: ———---- eq. (11b), − − − − almost complete coincidence, in the maximum radius accurate to $0.9R_k$; **b)** Intensity of mass gas injection Eq. (12-13) providing the proximity of non-stationary and stationary caverns at uniformly accelerated motion at $\sigma_a = 0.02$, $\kappa_c = 0.5$, $\bar{P}_\infty = 500$, $t^o_{co} = 15^o$, $\bar{t}_o = \bar{U}_o = 36.5$: ——— calculation by adiabatic $1/n = \kappa = 1.4$, ———-- calculation by isotherm $n = 1$.

4. Possibilities of cavity shape optimization during accelerated motion at a small angle to the horizon

The retention of the shape and size of the cavity close to stationary when moving at a small angle to the horizon at the constant pressure in the cavity can occur due to equality $\sigma_{nx} = \text{const}$ during the motion with the number of cavitation $\sigma = \bar{g}\theta$ for the stationary shape of the cavity. g - acceleration of gravity. The results of calculating the cavity behind the disk in this case - solution (14) are given in Figure 6.

$$\text{a) } \bar{R}^2 = 1 + \sqrt{\frac{2c_{do}}{k\mu}}\sqrt{2\bar{x}}\left[\bar{t} - \sqrt{2\bar{x}}\right] - \frac{\bar{g}\theta}{\mu}\bar{x}\left[\bar{t} - \sqrt{2\bar{x}}\right]^2 \qquad (14)$$



(a) (b)

Figure 6. Forms of an unsteady cavity at uniformly accelerated motion at small angle to the horizon at zero pressure in the cavity Eq. (14): **(a)** cavitator is moving, liquid is motionless: calculations for: $\bar{t} = 10$, $\bar{t} = 17$, $\bar{t} = 25$, $\bar{t} = 27$, -.-.-.- $\bar{t} = 28.23$ ———-- $\bar{t} = 30$, $\bar{t} = 40$; **(b)** cavitator is motionless, liquid is moving: − − − − **1 -** $\bar{t} = 10$, **2 -** $\bar{t} = 17$, **3-** $\bar{t} = 25$, **4 -** $\bar{t} = 27$,: -.-.-.-: **5 -** $\bar{t} = 28.23$: ———--- **6 -** $\bar{t} = 30$, **7-** $\bar{t} = 40$, -•-•- stationary cavity $\bar{t} = 100$.

Conclusions

The possibilities of controlling the cavity when gaining speed are determined, which provides a more favorable mode of motion in the cavity with the constancy of its sizes and shape , close to stationary.

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