Numerical Simulation of Tip Vortex Cavitation Inception

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Abstract: Tip vortex cavitating flow is known as challenging to study. The objective of this paper is to investigate the effective parameters in numerical modelling of tip vortex cavitation (TVC) inception through the comparison of three different models. The models are (1) a commonly used homogenous mixture model, in which inception is based on pressure drop criterion; (2) a Lagrangian bubble model, in which cavitation is initiated from free nuclei in the liquid; and (3) a hybrid Eulerian-Lagrangian model, in which the cavities are initiated based on the pressure drop criterion, but the growth of initially small cavities are modelled using the more accurate Lagrangian equations. The simulations are conducted on the tip vortex flow around an elliptical foil. The results show that the commonly applied pressure drop assumption is not a sufficient criterion for cavitation inception. Also, it is seen that the water quality and nuclei transport towards the vortex core influence the cavity pattern at inception.

Keywords: Tip vortex cavitation; Cavitation inception; Lagrangian bubble model

1. Introduction

As the flow passes over a lifting wing with finite span, due to the pressure difference between the upper and lower surfaces, the flow is highly three dimensional at the tip region and a vortex is created. Due to the swirling nature of the vortex, the pressure in the vortex core is lower than in the surrounding [1]. Therefore, in the cavitating case, it is expected that cavitation inception occurs in the vortex core. Reliable estimation of cavitation inception can be an issue with the commonly used numerical models in engineering applications (e.g. marine propellers). For fully developed cavitation, current industrial standard approach with a mixture representation of the vapour and liquid works well, but it is not sensitive enough for cases where inception and water quality are important features, such as in absolute determination of cavitation bucket and in model scale experiments of modest cavitation. In an earlier study it was shown that since the common homogenous models do not sufficiently take into account bubble inertia and ignore the effects of dissolved gas pressure as well as surface tension, they cannot resolve vapour structures at small scales and they fail to reliably predict cavitation inception [2,3]. In TVC experiments also, it is observed that the cavitation is intermittent at inception depending on how nuclei are transported into the core of the tip vortex. The inception dependency on the nuclei is also confirmed in a (simplified) numerical study [1]. To improve the prediction, Lagrangian bubble models are proposed, since they can consider the ignored effects in the homogeneous mixture models and it is also possible to take into account the water quality. For instance, Cheng et al [4], used a Lagrangian bubble model to include the dissolved gas pressure effect in the TVC inception, although in the applied model the surface tension effect and a complete description of bubbles inertia are still missing. Recently a hybrid mixture-bubble model has been developed in which small cavity structures are represented in a sub-grid Lagrangian modelling of discrete bubbles while the homogenous mixture approach is still used for the fully developed cavitation [3, 5]. In this coupled Eulerian mixture - Lagrangian bubble model, the cavitation is still initiated by the mixture model when the pressure falls below the saturation pressure. However, at the inception phase the small cavities are transformed to the Lagrangian framework. If Lagrangian cavities later grow enough that can be sufficiently represented by the mixture model, they are transformed back to the Eulerian framework. The application of the hybrid model to cavitating flow around a bluff body and comparing with experiment has shown significant improvements in inception prediction. The improvement includes both predicting the correct inception point and avoiding unrealistic inception that is predicted by the homogenous model at boundary layer separation. In this study,

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we model the TVC inception around an elliptical foil and compare the performance of a common mixture model with the hybrid model and a pure Lagrangian model. In the pure Lagrangian model, cavitation starts from free bubble nuclei distributed in the liquid flow and the effect of nuclei transport to the core of the tip vortex is investigated. The objective is to understand the limitations and capabilities of each model and to study the effective parameters at inception of tip vortex cavitation. The comparison between the Eulerian and hybrid models show that, despite the prediction of cavitation inception using the homogenous mixture model, the pressure drop assumption is not a sufficient criterion for cavitation inception. Also, the pure Lagrangian model results give the possibility to investigate the water quality effect and show how the nuclei transport towards the vortex core can influence the cavitation inception pattern.

2. Numerical Models and Geometry

In all of the applied models, the general continuity and Navier-Stokes equations are solved to calculate the main flow field and the difference between the models is in the representation of the vapour field.

2.1 Homogenous mixture model

In this approach, the cavity interface is modelled by solving a transport equation for the liquid volume fraction, α, as

$$\frac{\partial \alpha}{\partial t} + \frac{\partial (u_i \alpha)}{\partial x_i} = \frac{\dot{m}}{\rho_l}, \tag{1}$$

where u_i is the velocity vector and ρ_l is the liquid density. Also, $\dot{m}$ is the mass transfer rate at the interface, which is estimated using the Schnerr-Sauer model [6] of OpenFOAM. In this model, via the mass transfer source term, the cavitation is initiated in regions with pressure values smaller than the saturation vapour pressure. The mixture density, ρ_m, and dynamics viscosity, μ_m, are updated at each time step based on the liquid volume fraction as

$$\rho_m = \alpha \rho_l + (1 - \alpha)\rho_v, \qquad\qquad \mu_m = \alpha \mu_l + (1 - \alpha)\mu_v \tag{2}$$

2.2 Lagrangian bubble model

In this model, instead of representing the vapour structures by solving a transport equation (eq. 1), the cavities are transported as Lagrangian (parcels of) bubbles by solving bubble equation of motion. Then the liquid volume fraction, α, is calculated based on the bubbles' volume fraction in each computational cell. Also the bubble growth is modelled using an improved form of the Rayleigh-Plesset equation, which takes into account the local pressure effect on bubble dynamics [7]. In this model the vapour cavities are initiated from free nuclei in the flow field. The free nuclei are injected upstream of the elliptical foil as uniformly distributed bubbles with diameters of 5 & 50 μm (two conditions) and in initial mechanical equilibrium with the surrounding liquid. As the bubbles move downstream, due to the flow forces (especially the pressure gradient force) some of them are sucked towards the tip vortex core, and if the core pressure is low enough, the bubbles grow and cavitation is initiated.

2.3 Hybrid Eulerian mixture - Lagrangian bubble model

This model is developing by coupling of the Eulerian mixture model with the Lagrangian bubble model. In the hybrid mixture-bubble model, the small cavity structures are represented in a sub-grid Lagrangian modelling of discrete bubbles while the homogenous mixture approach is used for the large scale cavities. Here, cavitation is initiated in low pressure regions, via the mass transfer source term of eq. (1), and using the Eulerian homogenous mixture model. However since the initial cavities are at the sub-grid length scale, they are transformed to the Lagrangian framework to improve the prediction accuracy of the growth of small bubbles. In other words, here the small cavities at the inception phase are modelled in the Lagrangian framework, similar to the pure Lagrangian model; however, instead of initiating the cavities from free nuclei, the small cavities are initiated in the Eulerian framework and then they are transformed to the Lagrangian framework. In the current simulations of the hybrid model, the transformation of Lagrangian bubbles to Eulerian cavities does not happen, as the model parameters (defined in [2, 3]) are set in a way that small cavities are represented only in the Lagrangian framework.

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The governing equations are solved in the open source C++ package OpenFOAM (see [3, 5, 7] for further details). The flow domain and the boundary conditions are depicted in Figure 1. For domain discretization the earlier mesh named as P1S1 in [1] is used, which consists of 8.3 M cells with more refinements at the tip vortex region.
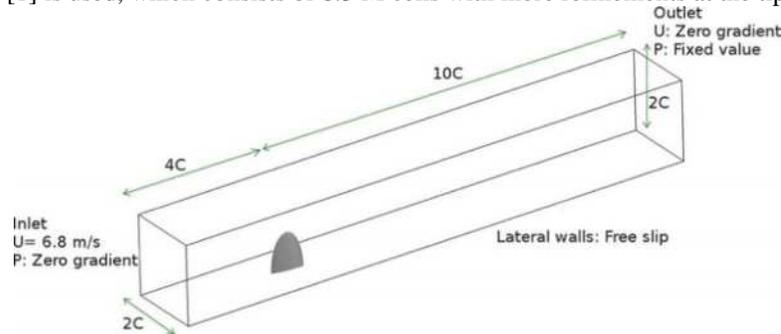

Figure 1. Elliptical foil and boundary conditions. The flow is from left to right.

3. Results

In this section the results of the three models in the prediction of cavitation inception are compared with each other. It should be mentioned that the results for the Eulerian mixture model are not new and they are extracted from [1]. Asnaghi et al. [1] investigated the effect of different numerical settings and grid resolution and showed that the Eulerian model predicts TVC inception at cavitation number of $\sigma = 4.2$. Figure 2(a) shows the obtained averaged vapour structure for this case. Here, the cavities are generated at the vortex core, where the flow has minimum pressure. If we apply the hybrid model for the same case, the initial sub-grid cavities are transformed into Lagrangian bubbles and the growth of these bubbles are calculated using the more accurate Rayleigh-Plesset equation which takes in to account the surface tension force, bubble inertia and dissolved gas effects. As shown in Figure 2(b), in this case the bubbles do not grow and the flow does not cavitate. It is worth mentioning that the hybrid model was applied to lower cavitation numbers, such as σ =3.5 & 2.6, and the bubbles yet did not grow enough to initiate cavitation. For further analyses, detailed experimental data are needed to investigate at which cavitation number and water quality the TVC inception should occur, however, from the comparison of the two models in Figure 2, it can be concluded that the pressure drop assumption is not a sufficient criterion for the estimation of cavitation inception.
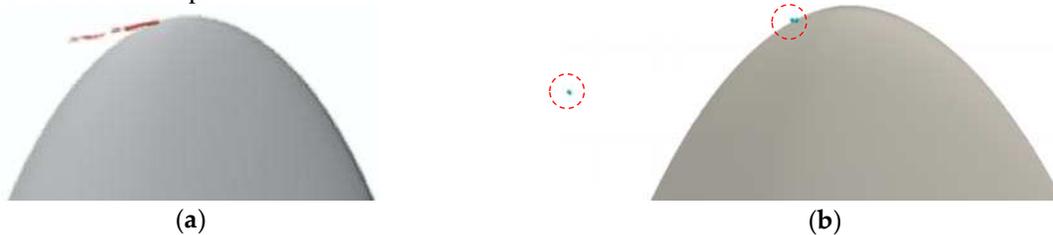

Figure 2. TVC inception at $\sigma = 4.2$: (**a**) Eulerian mixture model [1]; the red structure is the iso-surface of $\alpha = 0.9$. (**b**) Hybrid model result; the small blue bubbles are enlarged by 400R and enclosed by dashed circles.

In the pure Lagrangian model, free nuclei are injected from a 2D plane at upstream which is perpendicular to the main flow direction. To reduce the computational cost, the bubble parcels are injected only at the small effective region around the tip vortex. In Figure 3, it can be seen that the bubbles are transported towards the vortex core in helical paths and they have larger number density around the vortex centre, which is depicted by a red line in the figure. The helical paths around the vortex core are also visible in the corresponding vapour pattern in Figure 4, obtained from averaged bubbles volume fraction. The flow cannot be considered as cavitating for the initial nuclei size of 5 μm as the vapour volume fraction is very low (10^{-4}). However, TVC TVC inception occurs when the initial free stream nuclei diameter is 50 μm. In this case the bubble diameter

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can sometimes grow to a few centimeters. In addition to the water quality effect in inception, Figure 4 also shows a different vapour pattern, in which the initial bubbles move in the longer helical path rather than straight central line of the vortex (Figure 2). Such a difference and its consequent effects can be the subject of a future study where all models results are compared with detailed experimental data.

| (a) | (b) |

Figure 3. Nuclei transport towards the vortex core: (**a**) top view (**b**) front view. The bubble nuclei are enlarged by 150R and the red line depicts the vortex central line. The nuclei have initial diameter of 5 μm.
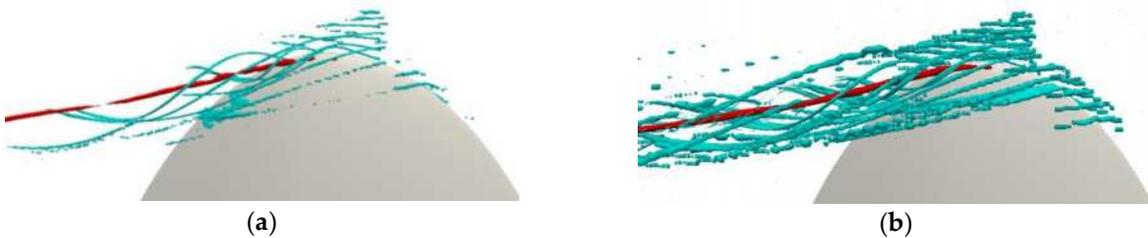

| (a) | (b) |

Figure 4. TVC inception using the pure Lagrangian model with free nuclei: (**a**) iso-surfaces of $\alpha = 0.9999$ for initial nuclei diameter of 5 μm (**b**) iso-surfaces of $\alpha = 0.99$ for initial nuclei diameter of 50 μm. The red line depicts the vortex central line.

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